2-edge-hamiltonian connectivity and the Thomassen conjecture

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joint work with
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Abstract

Let G be a graph and $E^+(G) = \{xy : x, y \in V(G)\}$. For $X \subset E^+(G)$ set $G + X = (V(G), E(G) \cup X)$. We say, that G is 2-edge-hamiltonian connected if for any $X \subset E^+(G)$ of size two $G + X$ has a hamiltonian cycle containing X. We show, that every 4-connected line-graph is hamiltonian if and only if every 4-connected line-graph is 2-edge-hamiltonian connected. If a graph G is 2-edge-hamiltonian connected then G is 4-connected. If the Thomassen conjecture is true then a line-graph G is 2-edge-hamiltonian connected if and only if G is 4-connected and thus it would be possible to decide about 2-edge-hamiltonian connectivity of line-graphs in polynomial time!